

Errors analysis in Understanding Transformation Geometry through Concept Mapping

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ABSTRACT

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The concept map is a chart that illustrates an understanding of a particular series. In the concept map, there is a relationship between concepts in a certain material according to the conceptual that is in the mind of the person who made it. This research emphasizes the analysis of errors, namely conceptual errors, by involving the construction of concept maps. The purpose of this research is to identify the conceptual errors of students in understanding the Transformation Geometry material through concept mapping. The subjects were IKIP Budi Utomo Malang students who were taking the Transformation Geometry course. This research was descriptive qualitative research. The instrument used was the concept map of geometry transformation made by students. Based on the concept map made by students, there are some errors, which are as follows. The error of low ability students in constructing concept maps is that they cannot give proper connections between materials and include examples of questions that should not need to be included in the concept map. The error of medium ability students is that they do not provide a clear description of each material. High ability students do not have any errors but they are less similar in giving explanations to each material.

Introduction

Mathematics is a subject that emphasizes more on concepts including the Transformation Geometry material. The contents of the Transformation Geometry material at the lecture level are deeper than in high school. Students already have the concept of this material when studied in high school. Therefore, students should be able to connect concepts that have been thought in high school with new concepts learned in this lecture. But many students have difficulty understanding the concept. As stated by Ruseffendi (2006) that there are many students are not able to understand even in the simplest parts and there are many mistakes or errors in understanding concepts during learning mathematics. The facts show that there are many conceptual errors made by students of Mathematics Education Study Program IKIP Budi Utomo Malang in understanding concepts of transformation geometry.

Errors in learning mathematics can be divided into several types. Newman divides errors into five, namely errors in decoding, comprehension, transformation, process skill, and encoding (Utami, 2016). Nolting (2011) divides errors into three, namely conceptual errors, misunderstanding problems, and careless errors. Riccomini (2016) divides errors into four, namely conceptual errors, procedural errors, factual errors, and carelessness. Based on the various classifications, the research is only emphasized on conceptual errors which improvements in the understanding of concepts are one of the focuses in learning mathematics.

Errors are an inevitable part of learning mathematics. Errors arise because of interactions between the mathematics features, the learning process, and social practices (Brodie, 2014). Furthermore, Sulistyorini (2017) states that errors are an inseparable part of learning mathematics. In line with this, Ingram, Baldry, & Pitt (2013) also states that errors are important in learning mathematics, both for students and teachers. From the perspective of students, making errors is the basis for constructing a concept. From the teacher's perspective, errors can be used as a basis for knowing how students understand mathematical concepts.

The strategy that can be used to repair errors is to conduct error analysis in learning mathematics. Error analysis is an effective step in overcoming misunderstandings and allowing students to reflect on their learning (Rushton, 2018). Error analysis also helps students in reflecting on their problem-solving skills (Fitriani, Turmudi, & Prabawanto, 2018).

Error analysis is not only beneficial for students but also has a positive impact on teachers. Error analysis is an inseparable part of the knowledge possessed by teachers (Sapire, et al, 2016). It means that how teacher takes action related to students' mistakes and depends on how deep knowledge and concepts of teachers that related to these errors. Also, error analysis can be a reference for teachers in choosing strategies, models or media learning that are used to reduce students' errors (Fitriani, Turmudi, & Prabawanto, 2018).

Shalem, Sapire, & Sorto (2014) explain that six criteria need to be considered by teachers in conducting error analysis. The first criterion is the procedural understanding of the correct answer that emphasizes the quality of the teacher's procedural explanation. The second criterion is the conceptual understanding of the correct answer that emphasizes the quality of the teacher's conceptual explanation. The third criterion is awareness of errors that emphasizes the teacher's explanation of actual mathematical errors not on the reasoning of students. The fourth criterion is the diagnosis of students' reasoning related to students' errors. The fifth criterion is the use of relationships in everyday life in the explanation of errors. And the sixth criterion is various explanations related to errors that emphasize the offering of various alternative explanations in correcting students' errors. Meanwhile, according to Brodie (2014), in error analysis, three important aspects are emphasized, namely identifying, interpreting and engaging with students' errors.

Efi (2016) said that mind mapping can be used to connect the student's idea in mind with new concepts they get. Furthermore, Wang (2019) said that concept mapping can help students to understand the correlation between important concepts.

The concept map is a chart that illustrates an understanding of a particular series. The use of concept maps can be considered as one of the initial efforts in identifying errors. According to Dahar (1998) concept maps can be used for specific purposes, including (1) to investigate something that learners know, (2) as one of the tools to know how to learn, (3) reveal the conceptual errors, and (4) can be used as evaluation tools. Concept maps are tools that can be used to find out what students have understood. Students actively think of the relationship with their concepts with new concepts that are received through the construction of concept maps. The concept map can encourage students' curiosity in learning materials. Moreover, mathematical concepts are almost all related. The explanation before emphasizes that concept maps are very useful for detecting students' conceptual errors. The acquisition of new knowledge is a function of the cognitive structure that must link their concepts with new concepts that already learned.

Previous studies such as Sulistyorini, (2017a) emphasized the analysis of errors in solving Euclid's Geometry problems by considering the assimilation and accommodation of the concepts being learned. Sulistyorini (2017b) emphasizes the analysis of errors in solving differential equations by considering scaffolding in correcting students' errors. Utami (2016) emphasizes the analysis of errors based on Newman on geometry concepts. While this research emphasizes the analysis of errors, namely conceptual errors, by involving the construction of concept maps. Therefore, this research focuses on describing student conceptual error by involving the construction of concept maps. Based on this description, teacher

Research Methods

This type of research was a qualitative descriptive study. In this study using qualitative data to identify the conceptual errors of students in understanding the Transformation Geometry material through concept mapping. Qualitative data in this study are student work in the construction of concept maps.

The location chosen in this study is IKIP Budi Utomo Malang. The subject of research were students who take the Transformation Geometry course. The subjects chosen were six people, two students with high mathematical ability, two students with medium mathematical ability, and two students with low mathematical ability.

Data analysis consists of the stages of data reduction, data presentation and conclusion drawing (Sugiyono, 2017). Data reduction was the stage of separating the data needed and not needed in research based on the work of concept maps and interviews with subjects. Data presentation was the stage of preparing relevant data based on the results of data reduction by describing narratively and analyzed in depth. The data presentation stage refers to three important aspects in error analysis according to Brodie (2014) which consists of identifying, interpreting and engaging with students' errors. However, this research was

only limited to identifying errors. The conclusion was the last step to obtain a description of students' conceptual errors in understanding the geometry of transformation. Data validity testing was done by triangulation of sources. Source triangulation was shown by selecting two subjects for each high, medium and low mathematical ability.

Results and Discussion

Students were asked to construct a concept map of the geometry of transformation concepts which includes translation, rotation, reflection, and dilation. Then randomly selected two concept maps of subjects with low mathematical ability, two concept maps of subjects with medium mathematical ability, and two concept maps of subjects with high mathematical abilities. For subjects with low ability were given SR1 and SR2 code. For subjects with medium ability were given SS1 and SS2 code. For subjects with high abilities were given ST1 and ST2 code. Errors identification for each subject is presented below.

First Low Ability Subject (SR1)

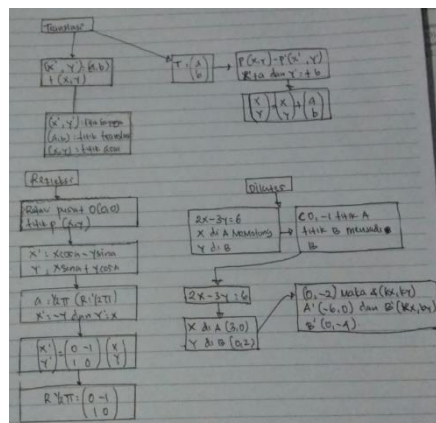


Figure 1. Concept Map of SR1

The conceptual error of SR1 was not knowing the relation between rotational, reflection, dilation, and translation. Concept maps were made on one sheet but each material was separate. In the concept map, the material was separated into three types, namely translation, reflection, and dilation. As for rotation material was included in reflection material. Another conceptual error was reflective and rotation was connected but this relationship was not right. This error was indicated by arrow that start from reflection material and then end up in rotation material. The rotation material was written down some rules that apply to the rotation material. The dilation material was written as an example problem involving certain numbers.

Second Low Ability Subject (SR2)

Concept maps were constructed into several sheets. SR2 subjects separate each material on a different sheet. In the concept map, the material was separated into three types, namely

translation, reflection, and rotation. Thus, it can be said that the subject was not able to construct a connection between materials.

On translation material written various types of objects that were translated. For each type, the right formula was written and examples of problems were outlined along with ways to solve the problems.

In reflection material, the subject divided reflection into two, namely reflection in the Cartesian plane and determines general formula in reflection. Based on an analysis, the subject divided this according to procedures that have been taught in class. After dividing reflection into two, the subject wrote a general formula. The subject also gave a description consist of an example involving a number.

In rotation material, the subject also divided rotation into two, namely rotation in the Cartesian plane and determined general formula in rotation. The subject also explained for each section by writing a general formula and sample questions with the answers.

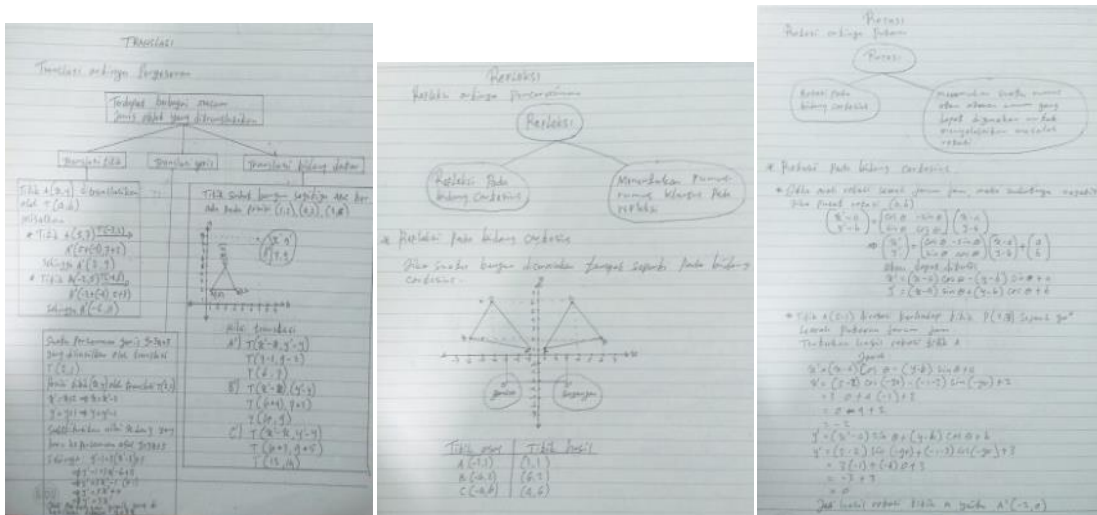


Figure 2. Concept Map of SR2

First Medium Ability Subject (SS1)

The subject constructed a concept map about the transformation consisting of isometry, translation, reflection, rotation, and dilation. In each material written any cases that occurred. But there was an error in writing hyphen. The first case was revealed to be the second case, the second case was revealed to be the third case, and so on. Though each case should have the same position. In each case that was written did not provide a more detailed explanation.

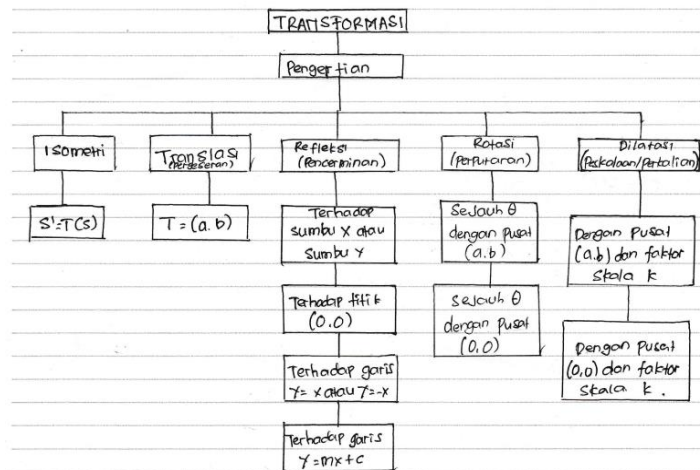


Figure 3. Concept Map of SS1

Second Medium Ability Subject (SS2)

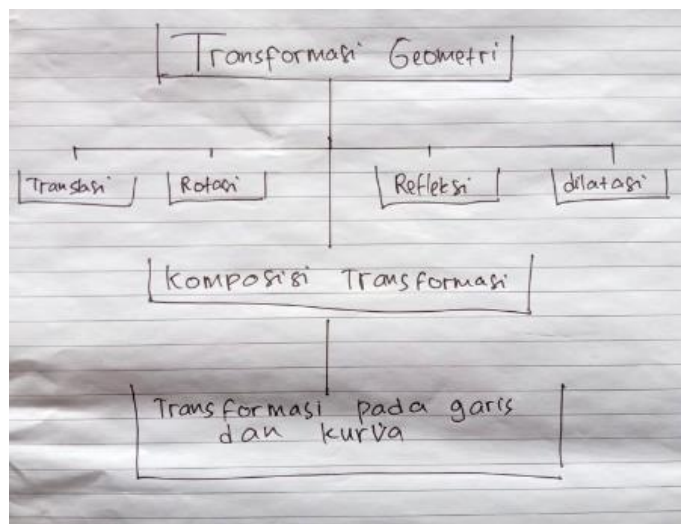


Figure 4. Concept Map of SS2

The concept maps consist of translation, rotation, reflection, dilation and transformation composition. Even though the composition of the transformation should lie after translation, rotation, reflection, and dilation. If seen based on the position of the writing on SS2' concept map, the composition of the transformation is indeed beneath translation, rotation, reflection, and dilation. But if seen based on the hyphen on SS2' concept map, the composition of the transformation is parallel with translation, rotation, reflection, and dilation. After the composition of the transformation, SS2 continued the line connecting it to the transformation on the line and curve. The subject did not specify what is written on the concept map.

First High Ability Subject (ST1)

ST1 made a concept map by writing all the material in a complex way. Even prerequisite material before studying translation, rotation, reflection, and dilation was written. On this concept map made, the subject was able to distinguish between each material. The subject also wrote meaning or case that occurs for each translational, rotational, reflective, and dilated material. On translational material written the type and nature. Reflection material was written on a variety of cases that occur. On the rotation material was written the type of case that occurs. In the dilated material was written the meaning of the change.

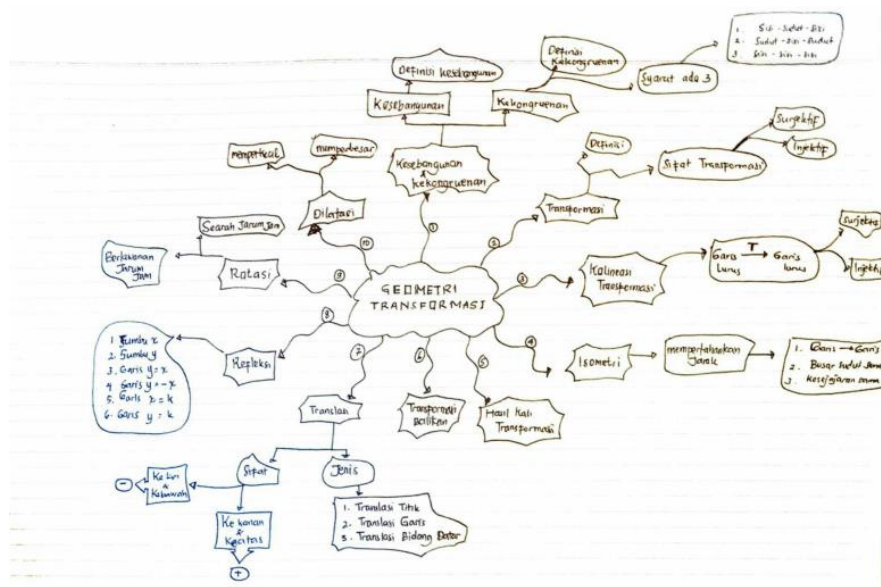


Figure 5. Concept Map of ST1

Second High Ability Subject (ST2)

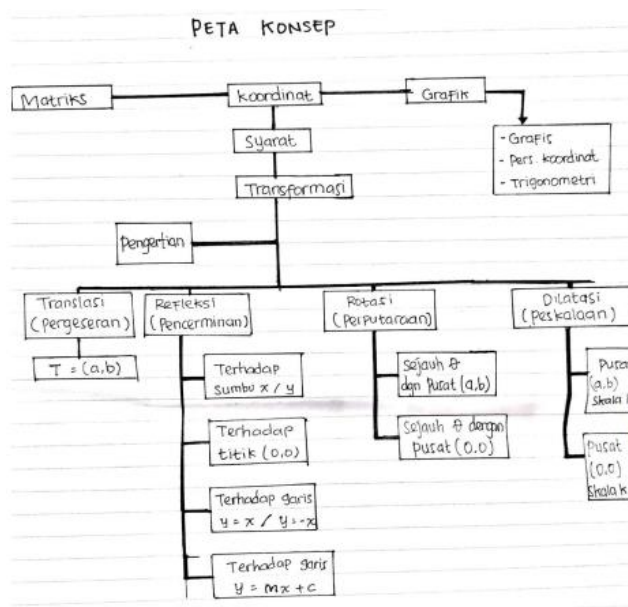


Figure 6. Concept Map of ST2

ST2 made a concept map by writing all the prerequisite material needed in studying translational, rotational, reflective, and dilated material. Prerequisite materials needed being matrices and graphs. Matrices and graphs related to coordinates required for transformation. Then transformation was divided into translation, rotation, reflection, and dilation. In each material, the meaning was written in the Indonesian term. On translational material was written translational symbols. On the material reflection, rotation, and dilation was written a variety of cases that occur.

Based on the exposure, here is a description of each subject. SRI subjects could not link between concepts correctly and on the concept map that was made included examples of problems involving certain numbers. While the subject SR2 separates between material and each material is included with example problems. For SS1 subjects, it is appropriate to provide links between concepts, but there is a breakdown in each material that is incorrect. While SS2 subjects are right in making connections between materials, there are a few things that are not quite right. Whereas the subject of ST1 made a concept map by writing all the material along with the prerequisite material. The subject of ST2 creates a concept map by providing links between concepts appropriately and including prerequisite material as well.

The similarity between the two subjects that have the same mathematical ability will be described. On the subject SR1 and SR2 have in common that is not making a proper relationship between translational material, reflection, rotation, and dilation. Also, on the concept map made the subjects SR1 and SR2 both provide an example of a number.

SS1 and SS2 have in common that is making concept maps by providing relationships or linkages in each translational, reflection, rotation, and dilation material. The relationship between these four materials is precisely made.

ST1 and ST2 have in common that is both writing prerequisite material to understand translational, reflection, rotation, and dilation material. Besides, ST1 and ST2 describe the meaning or meaning of each material.

Based on these similarities, it can be said that the concept maps of the geometry of transformations made by subjects with low mathematical abilities do not have a proper relationship between materials. Subjects with low mathematical abilities also give examples of numbers. Subjects with medium mathematical abilities provide relationships between materials appropriately but do not provide clear descriptions. Subjects with high mathematical abilities construct proper connections between materials and give meaning to each material and provide prerequisite material for understanding translational, reflection, rotation, and dilation material. However, each subject who has high ability does not provide the same explanation.

Based on these descriptions, it can be identified as follows. Subjects with low mathematical abilities cannot provide connections between materials and on concept maps include examples of problems that should not need to be included in. Subjects with medium mathematical abilities can provide links between materials but do not provide clear descriptions. Subjects with high mathematical abilities can provide proper connections

between materials, give meaning to each material, and even provide prerequisite material to understand translational, rotation, reflection and dilation material, but are not similar in providing explanations for each material.

Based on the description, it is known that students with high mathematical abilities can understand the connections between concepts in each material and even still have prior knowledge as a prerequisite for understanding transformation geometry. While students who have medium and low mathematical abilities do not remember the prerequisite material. This is consistent with the statement of Maifa (2019) that errors can occur because students do not understand prior knowledge well.

Conclusion

Based on the results of the analysis of research data and the discussion that has been described, the following conclusions can be drawn. The error of low ability students in constructing concept maps is that they cannot give proper connections between materials and include examples of questions that should not need to be included in the concept map. The error of medium ability students is that they do not provide a clear description of each material. High ability students do not have any errors but they are less similar in giving explanations to each material. This description can be used by teachers as a reference for designing learning that is appropriate for learners with low, medium, and high abilities. Further research can be extended to interpreting and engaging in students' errors, not only for identifying errors.

Bibliography

- Brodie, K. (2014). Learning about Learner Errors in Professional Learning Communities. *Educational Studies in Mathematics*, 85(2), 221–239. <https://doi.org/10.1007/s10649-013-9507-1>.
- Dahar, R. W. (1998). *Teori-Teori Belajar*. Jakarta: Departemen Pendidikan dan Kebudayaan.
- Efi, Darsikin, & Saehana, S. 2016. Pengaruh penggunaan Metode Mind Mapping terhadap Keterampilan Berpikir Kreatif Siswa Kelas X SMA Negeri 2 Balaesang. *Jurnal Pendidikan Fisika Tadulako*, 5 (3), 3-6.
- Fitriani, H. N., Turmudi, T., & Prabawanto, S. (2018). Analysis of Students Error in Mathematical Problem Solving Based on Newman's Error analysis. In *International Conference on Mathematics and Science Education*.
- Ingram, J., Baldry, F., & Pitt, A. (2013). The influence of how teachers interactionally manage mathematical mistakes on the mathematics that student experience. Antalya. Retrieved from http://cerme8.metu.edu.tr/wgpapers/WG9/WG9_Ingram.pdf.
- Maifa, T. S. (2019). Analisis Kesalahan Mahasiswa dalam Pembuktian Transformasi Geometri. *Jurnal Riset Pendidikan Dan Inovasi Pembelajaran Matematika*, 3(1), 8–14.

<https://doi.org/10.26740/jrpipm.v3n1.p8-14>.

- Nolting, P. D. (2011). *Math Study Skills Workbook Fourth Edition*. Cengage Learning.
- Riccomini, P. J. (2016). *How to use math error analysis to improve instruction*. Retrieved from <http://files.ernweb.com/erroranalysis.pdf>
- Ruseffendi, E. T. (2006). *Pengantar kepada Guru Membantu Mengembangkan Kompetensinya dalam Pengajaran Matematika untuk Meningkatkan CBSA*. Bandung: Tarsito.
- Rushton, S. J. (2018). Teaching and Learning Mathematics through Error Analysis. *Field Mathematics Education Journal*, 3(4). <https://doi.org/https://doi.org/10.1186/s40928-018-0009-y>.
- Sapire, I., Shalem, Y., Wilson-Thomson, B., & Paulsen, R. (2016). Engaging with Learners' Errors when Teaching Mathematics. *Pythagoras*, 37(1), a331. <https://doi.org/http://dx.doi.org/10.4102/pythagoras.v37i1.331>.
- Shalem, Y., Sapire, I., & Sorto, M. A. (2014). Teachers' explanations of learners' errors in standardised mathematics assessments. *Pythagoras*, 35(1), a254. <https://doi.org/http://dx.doi.org/10.4102/pythagoras.v35i1.254>.
- Sugiyono. (2017). *Metode Penelitian Kuantitatif, Kualitatif, dan R&D*. Bandung: Alfabeta.
- Sulistiyorini, Y. (2017a). Analisis Kesalahan dan Scaffolding dalam Penyelesaian Persamaan Diferensial. *Kalamatika*, 2(1), 91–104. <https://doi.org/https://doi.org/10.22236/KALAMATIKA.vol2no1.2017pp91-104>.
- Sulistiyorini, Y. (2017b). Error Analysisin Solving Geometry Problem on Pseudo-Thinking's Students. In *International Conference of Mathematics Education (INCOMED)*. Malang. Retrieved from <https://download.atlantis-press.com/article/25893805.pdf>.
- Utami, A. D. (2016). Tipe Kesalahan Mahasiswa dalam Menyelesaikan Soal-Soal Geometri Berdasar Newman's Error Analysis (NEA). *JIPM (Jurnal Ilmiah Pendidikan Matematika)*, 4(2), 85–92. <https://doi.org/10.25273/jipm.v4i2.842>.
- Wang, Shun-Ho. 2019. Instruction Design and Strategy of Concept Mapping. In *5th International Conference of Economics, Management, Law, and Education (EMLE 2019)*. Advances in Economics, Business, and Management Research Volume 110. Hal 1195-1198.